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Slow drift of a floating cylinder in narrow-banded beam seas

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For a long cylinder floating on the sea surface, incident sea waves with a narrow frequency band excite body oscillations of short and long periods. Depending on the stiffness of the mooring system, the body displacement of the long-period motion can be comparable with, or even greater than that of the short-period oscillations. By combining the asymptotic methods of multiple scales and inner and outer expansions, we describe an essentially analytical theory for slow sway of both small and large amplitudes. Besides showing results for various quasi-steady and transient incident waves for a rectangular cylinder, we examine the effect of the gap between the keel of the body and the sea bottom. It is found in particular that a small gap can enhance moderate resonance by blocking the flow due to long waves and increase the apparent mass of the cylinder. Real-fluid effects are not included.

1. Introduction

Moored vessels and offshore platforms are often subject to seas with narrow-banded spectra. Since their mooring systems may have natural frequencies of horizontal plane motions (sway, surge, yaw) in the order of 0.01 Hz, these vessels can be excited by long-scale fluid motions associated with the modulational periods of incident wave groups.

In a regular (unmodulated) wave train, the steady drift force, which is second order in wave slope, can be computed from the first-order (linearized) solution. In irregular waves, Newman (1974) has found that the slow drift force can be written as a quadratic transfer function of the wave components. The coefficients of this function can be expanded as functions of the difference frequencies. He suggests that for small frequency differences the coefficients can be approximated by their values at zero difference – thus the slowly varying drift force is almost as simple as the steady drift force. In many other papers, the slow motion is found as a part of the complete and complicated second-order theory, see Pinkster (1976) or Ogilvie (1983) for a survey. Triantafyllou (1982) has observed that for finite depth the slow potential is of first order, and used a multiple time expansion to study large-amplitude ($O(1)$) slow motion. This technique was also employed by Molin & Bureau (1980). In these papers, the computational task is considerable. Reasoning that slow motions are associated with long waves, Agnon & Mei (1985) employed multiple scales in both time and space to study a rectangular block in beam seas, and examined second-order slow motions analytically.

In this paper we extend our earlier analysis to a two-dimensional floating body in beam seas. In §2 the problem is formulated. Boundary conditions on the body are given in §3. In §§4–7, small slow sway is described: fast motion is in §4 and slow motion in §§5 and 6, and examples for a narrow-gap geometry are given in §7. Sections 8–10 describe large slow sway: fast motion is studied in §8, slow motion in §9 and examples are given in §10. By combining the methods of multiple scales and matched asymptotics, analytical results are given for the transient evolution of slow drift motion and the radiation of long waves. Although the present theory is explained only for a rectangular cylinder allowed to sway, extensions to arbitrary cross-section and to three degrees of freedom require only known techniques of computation for the linearized part and involve no new principle. Results of such computations are presented.

2. Formulation

Under the usual assumptions of potential theory, the Laplace equation holds for the velocity potential $\Phi(x, z, t)$

$$\Delta\Phi = 0 \quad \text{in the fluid,} \tag{2.1}$$

where (x, z) are Cartesian coordinates, with the positive z-axis pointing vertically upwards, and t denoting time. Using g for gravitational acceleration, P for pressure and ρ for the fluid density, we have the Bernoulli equation:

$$-\frac{P}{\rho} = gz + \Phi_t + \tfrac{1}{2}|\nabla\Phi|^2. \tag{2.2}$$

Assuming zero pressure on the free surface at $z = \zeta$ and small wave steepness, we expand the free-surface boundary condition for Φ around the rest position of the free surface, and get

$$\Phi_{tt} + g\Phi_z = \left[-\tfrac{1}{2}(\nabla\Phi)^2 + \frac{1}{g}\Phi_t\Phi_{zt}\right]_t - (\Phi_x\Phi_t)_x + O[\Phi_x^3, \Phi_t^3] \quad (z = 0). \tag{2.3}$$

At the rigid horizontal bottom, the kinematic boundary condition is:

$$\Phi_z = 0, \quad z = -h. \tag{2.4}$$

Throughout this paper, the water depth is assumed to be comparable with the wavelength:

$$kh = O(1). \tag{2.5}$$

There are two small parameters associated with slowly varying small-amplitude waves. The first is the wave steepness ϵkA, where k is the central wavenumber and ϵA is the free-surface amplitude of the short wave, where $kA = O(1)$. The second parameter is the modulation ratio $\epsilon'\Omega/\omega$, where $\epsilon'\Omega$ is the frequency of modulation of the short wave, or equivalently, its frequency bandwidth, with $\Omega = O(\omega)$. For simplicity, we shall choose ϵ equal to ϵ', so as to render effects of dispersion and nonlinearity comparable.

In the near field of the body defined by $kx = O(1)$, evanescent modes are as important as the propagating modes; together they satisfy the boundary condition on the body. The lengthscale is $1/k$ in all directions but there are two timescales, $1/\omega$ and $1/\omega\epsilon$, the latter because of the slow modulation in the incident waves. It is well known that in an unobstructed sea, the envelope of propagating waves within

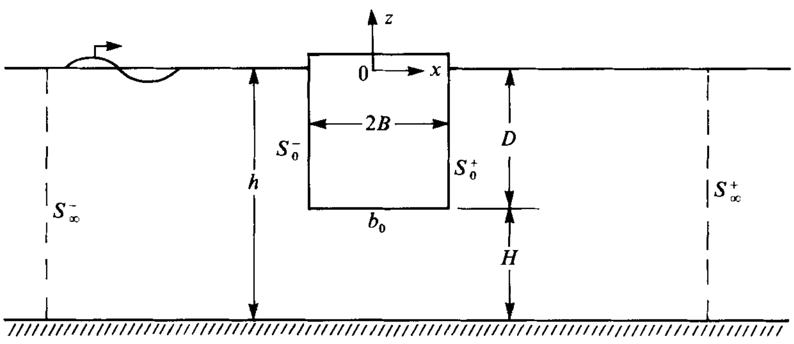

FIGURE 1. The near field of a two-dimensional problem.

a narrow frequency band of $O(\epsilon\omega)$ must be slowly varying in space with the lengthscale $O(1/\epsilon k)$. We define the far field by $|kx| = O(1/\epsilon)$ where only the propagating modes of the short waves are expected to be relevant. In the scheme of multiple-scale expansions, the neat field is described by the independent variables x, z, t_1 and the far field by x, z, t, x_1 and t_1 where

$$(x_1, t_1) = \epsilon(x, t). \tag{2.6}$$

This division into near and far fields enables us to disregard the long wave in the former and the evanescent modes in the latter; thus the analysis can be simplified.

3. Boundary conditions on the floating body

To continue the description of our approach, it is sufficient to focus on a two-dimensional problem of a horizontal cylinder in beam seas. At static equilibrium the axis of the cylinder lies on the y-axis. Waves are incident from $(x \rightarrow -\infty)$ (figure 1). For simplicity of presentation, we assume that the body has a rectangular cross-section and performs sway motion only, to the leading order, at both fast and slow timescales. Inclusion of all three modes of the fast motion involves no difficulty in principle and gives no qualitatively new feature in most cases. As will be reasoned shortly, sway is the most important of all slow modes of a cylinder. The mooring system is modelled by a linear spring.

The wave field is coupled with the motion of the body through boundary conditions on the body surface. Denoting the sway displacement by X, the exact kinematic boundary condition is, before introducing multiple scales,

$$\Phi_x = X_t \quad \text{on } S^{\pm} = \{x = X(t) \pm B; \quad -D \leqslant z \leqslant \zeta\}, \tag{3.1}$$

$$\Phi_z = 0 \quad \text{on } b = \{z = -D; \quad -B \leqslant x - X \leqslant B\}. \tag{3.2}$$

$S^{\pm}$ are the vertical sides of the body, which has breadth $2B$ and draught D (see figure 1). B and D are assumed to be comparable with the depth h which is of the order of the wavelength $2\pi/k$. We shall denote the boundaries of the body at rest by

$$S_0^{\pm} = \{x = \pm B; \quad -D \leqslant z \leqslant 0\}, \qquad b_0 = \{z = -D; \quad -B \leqslant x \leqslant B\}$$

and the mean position by averaging with respect to the short-wave period by

$$\bar{S}_0^{\pm} = \{x = \pm B + X_0(t_1); \quad -D \leqslant z \leqslant 0\}; \qquad \bar{b}_0 = \{z = -D; \quad -B \leqslant x - X_0(t_1) \leqslant B\}.$$

The exact dynamic boundary condition on the body is, before introducing slow coordinates,

$$MX_{tt}+KX = \int_{S^-} P\,\mathrm{d}z - \int_{S^+} P\,\mathrm{d}z, \tag{3.3}$$

where M is the mass of the body and K is the elastic constant of the mooring system. The right-hand side is the hydrodynamic force, where the pressure P is given by (2.2). As is well known, the driving force for the slow motion (zeroth harmonic) is $O(\epsilon^2)$.

Let us denote the time average with respect to $2\pi/\omega$ by an overbar. The magnitude of $\bar{X}$ depends on the mooring stiffness K. If the mooring is moderately weak so that $K = O(\epsilon)$, $\bar{X}$ must be $O(\epsilon)$ (small displacement) in order that the spring force should balance the hydrodynamic force. The mass of the body is $M = O(1)$† so that the inertia of the body for the slow motion is in general

$$M\bar{X}_{tt} = \epsilon^2 M\bar{X}_{t_1 t_1}, \tag{3.4}$$

which is $O(\epsilon^3)$ and negligible.

On the other hand, if the mooring is very weak so that $K = O(\epsilon^2)$, $\bar{X}$ must be $O(1)$ in order that the mooring force balances the slow-drift force. The body inertia is then $O(\epsilon^2)$ and is no longer negligible.

Some remarks on the magnitudes of other components of the body displacement are warranted here. For the slow heave Z and roll θ, the dominant terms of the restoring force and moment are due to buoyancy, the inertia term being $O(\epsilon^3)$. For heave the buoyancy force is

$$-2B\rho g Z = O(Z), \tag{3.5}$$

while the restoring moment for roll is

$$-gMm_c\theta = O(\theta), \tag{3.6}$$

where the metacentric height m_c is assumed to be $O(1)$. Since the forcing for slow motion is $O(\epsilon^2)$, we find that the amplitudes of the slow heave and roll, too, are $O(\epsilon^2)$, and much smaller than the slow sway. An exception is a floating body with a bottle neck at the water plane such as a semi-submersible whose water-plane area is very small. Thus slow sway is usually the most important mode of drift motion.

From now on it is convenient to examine separately the *near field*, within a few short waves from the body, and the *far field*, a few wave groups away from the body on either side.

We first describe in §§4–7 the case where the drift motion is of small amplitude $kX = O(\epsilon)$. Modifications for large amplitude $kX = O(1)$ is presented in §§8–10.

4. Slow drift of small displacement: fast motion

4.1. *The near field*

We first consider a stiff mooring $K = \epsilon K_1 = O(\epsilon)$, so that the body sway X, which is of order $O(\epsilon)$, can be expanded into harmonics as follows:

$$X = \sum_{n=1}^{\infty} \epsilon^n \sum_{m=-n}^{n} X_{nm} \exp(-\mathrm{i}m\omega t). \tag{4.1}$$

† In physical parameters $M = 2\rho BD$ by Archimedes' law.

Let us distinguish the potential Φ in the near field by ψ. At the first order, first harmonic, the short-wave potential in the near field ψ_{11} satisfies (2.1), and

$$\psi_{11_z} - \sigma\psi_{11} = 0 \quad \text{on } z = 0, \tag{4.2}$$

$$\psi_{11_x} = -\mathrm{i}\omega X_{11} \quad \text{on } S_0^{\pm}, \tag{4.3}$$

$$\psi_{11_z} = 0 \quad \text{on } b_0,\, z = -h, \tag{4.4}$$

$$-\omega^2 M X_{11} = -\mathrm{i}\omega\rho\left[\int_{S_0^-} \psi_{11}\,\mathrm{d}z - \int_{S_0^+} \psi_{11}\,\mathrm{d}z\right]. \tag{4.5}$$

In addition, ψ_{11} must satisfy the radiation condition. Formally, these equations are identical to the equations for the linear, time-harmonic problem of a rectangular cylinder swaying freely in regular waves. Many numerical methods can be used to solve this linear problem of diffraction and radiation. In particular one can determine the reflection and transmission coefficients R and T associated with the propagating modes:

$$\psi_{11} = a(t_1) f_0(z) \begin{Bmatrix} (\mathrm{e}^{\mathrm{i}kx} + R\,\mathrm{e}^{-\mathrm{i}kx}), & -kx \gg 1, \\ T\,\mathrm{e}^{\mathrm{i}kx}, & kx \gg 1, \end{Bmatrix} \tag{4.6}$$

where

$$f_0(z) = \frac{\surd 2 \cosh k(z+h)}{(h + \sigma^{-1}\sinh^2 kh)^{\frac{1}{2}}}, \tag{4.7}$$

with k being the positive real root of

$$\sigma \equiv \frac{\omega^2}{g} = k \tanh kh. \tag{4.8}$$

The first-order displacement amplitudes A of the short incident wave is related to the potential amplitude a by

$$A = \frac{2\mathrm{i}\omega a f_0(0)}{g}. \tag{4.9}$$

As an interesting special case to be examined further later, we consider the gap between the bottom and the body to be narrow:

$$h - D = O(\epsilon h). \tag{4.10}$$

In view of this assumption, flow in the gap is roughly uniform, forced by the pressure gradient:

$$-\frac{\partial P}{\partial x} \approx -\frac{\mathrm{i}\omega\rho[\epsilon\psi_{11}(B, -h) - \epsilon\psi_{11}(-B, -h)]}{2B} = O(\epsilon). \tag{4.11}$$

When multiplied by the gap width, this flow gives rise to an $O(\epsilon^2)$ flux. Its effect is that of a pair of oscillating sink and source of strength $O(\epsilon^2)$ and is negligible outside the gap. The potential outside the gap, ψ_{11}, is then given to $O(\epsilon)$ by the solution in Agnon & Mei (1985) for a sliding block, as if the gap did not exist.

4.2. *The far field*

Away from the body ($x_1 = O(1)$) the short waves and long waves have been analysed by Agnon & Mei (1985). We only recall that, up to $O(\epsilon^2)$, the far-field potential, which we denote by ϕ, is

$$\phi = \epsilon[\phi_{10} + (\phi_{11}\,\mathrm{e}^{-\mathrm{i}\omega t} + *)] + \epsilon^2[\phi_{20} + (\phi_{21}\,\mathrm{e}^{-\mathrm{i}\omega t} + *) + (\phi_{22}\,\mathrm{e}^{-2\mathrm{i}\omega t} + *)] + O(\epsilon^3). \tag{4.12}$$

At $O(\epsilon)$ the short-wave potential consists of the propagating modes only,

$$\phi_{11} = f_0(z)\,[Q^+(x_1, t_1)\,e^{ikx} + Q^-(x_1, t_1)\,e^{-ikx}], \tag{4.13}$$

where
$$Q^+ = Q^+\left(t_1 - \frac{x_1}{C_g}\right), \quad \text{and } Q^- = Q^-\left(t_1 + \frac{x_1}{C_g}\right), \tag{4.14}$$

in order for Φ_{21} to be solvable (see e.g. Mei 1983 p. 52). In order to match with the near field we require that

$$\left.\begin{aligned} Q^+ &= a(t_1 - x_1/C_g);\quad Q^- = Ra(t_1 + x_1/C_g) \quad (x_1 < 0),\\ Q^+ &= Ta(t_1 - x_1/C_g);\; Q^- = 0 \qquad\qquad\qquad (x_1 > 0).\end{aligned}\right\} \tag{4.15}$$

5. Slow drift of small displacement: slow motion in the far field

The long waves are associated with the zeroth harmonic of the potential ϕ_{10}, and of the surface elevation. Owing to the stretched coordinates, this first-order slow potential gives rise to second-order free-surface displacement

$$-\frac{1}{g}\frac{\partial}{\partial t}\epsilon\Phi_{10} = -\frac{1}{g}\epsilon^2\Phi_{10t_1} = \epsilon^2\zeta_{20}. \tag{5.1}$$

In the far field, $x_1 = O(1)$†, the governing equation for ϕ_{10} has been derived by Agnon & Mei (1985). From their (4.7) we can infer that

$$\phi_{10t_1t_1} - gh\phi_{10x_1x_1} = f_0^2(0)\,[\sigma^2 - k^2 - 2\omega k/C_g]\,[|Q^+|^2 + |Q^+|^2]_{t_1}. \tag{5.2}$$

In view of (4.15), (5.2) becomes

$$\begin{aligned}&\phi_{10t_1t_1} - gh\phi_{10x_1x_1}\\ &\quad = -f_0^2(0)\,[(k^2-\sigma^2)\,C_g + 2\omega k]\begin{cases}\dfrac{\partial}{\partial t_1}\{|a(t_1 - x_1/C_g)|^2 + |Ra(t_1 + x_1/C_g)|^2\}, & x_1 < 0,\\[2ex] \dfrac{\partial}{\partial t_1}|Ta(t_1 - x_1/C_g)|^2, & x_1 > 0.\end{cases}\end{aligned} \tag{5.3}$$

The right-hand side of (5.3) forces group-locked long waves which propagate at velocities C_g and $-C_g$; these are the inhomogeneous solutions to (5.3) without regard to boundary conditions. In addition, there are also free waves which propagate away from the origin at velocities $(gh)^{\frac{1}{2}}$ and $-(gh)^{\frac{1}{2}}$, and are solutions to the corresponding homogeneous equation. Formally, the entire solution can be written as

$$\phi_{10} = \left\{\begin{aligned}&\phi_{10}^{\mathrm{I}}(t_1 - x_1/C_g) + \phi_{10}^{\mathrm{R}}(t_1 + x_1/C_g) + \phi_{10}^{-}(t_1 + x_1/(gh)^{\frac{1}{2}}) && (x_1 < 0),\\ &\phi_{10}^{\mathrm{T}}(t_1 - x_1/C_g) + \phi_{10}^{+}(t_1 - x_1/(gh)^{\frac{1}{2}}) && (x_1 > 0),\end{aligned}\right\} \tag{5.4}$$

which must later be matched to the near field. The potentials of group-locked waves $\phi_{10}^{(\alpha)}$ (α = I, R, or T) can be straight forwardly obtained from (5.3) and written as

$$\begin{bmatrix}\phi_{10t_1}^{\mathrm{I}}\\ \phi_{10t_1}^{\mathrm{R}}\\ \phi_{10t_1}^{\mathrm{T}}\end{bmatrix} = \frac{f_0^2(0)}{gh/C_g^2 - 1}\left[(k^2 - \sigma^2) + \frac{2\omega k}{C_g}\right]\begin{bmatrix}|a(t_1 - x_1/C_g)|^2\\ |R|^2\,|a(t_1 + x_1/C_g)|^2\\ |T|^2\,|a(t_1 - x_1/C_g)|^2\end{bmatrix}. \tag{5.5}$$

The free waves $\phi_{10}^{\pm}$ are yet to be determined.

† Unless otherwise specified, all spatial coordinates are normalized by k^{-1} and time by ω^{-1}, when orders of magnitudes are mentioned.

For later matching with the near field, we shall need the inner expansion of (5.4) as $|x_1| \to 0$;

$$\left.\begin{aligned} \phi_{10} &\sim [\phi_{10}^{\mathrm{I}}+\phi_{10}^{\mathrm{R}}+\phi_{10}^{-}]_{x_1=0}+x_1\left[-\frac{\phi_{10}^{\mathrm{I}}}{C_{\mathrm{g}}}+\frac{\phi_{10}^{\mathrm{R}}}{C_{\mathrm{g}}}+\frac{\phi_{10}^{-}}{(gh)^{\frac{1}{2}}}\right]_{t_1} \quad (x_1<0),\\ \phi_{10} &\sim [\phi_{10}^{\mathrm{T}}+\phi_{10}^{+}]_{x_1=0}+x_1\left[-\frac{\phi_{10}^{\mathrm{T}}}{C_{\mathrm{g}}}-\frac{\phi_{10}^{+}}{(gh)^{\frac{1}{2}}}\right]_{t_1} \quad (x_1>0),\end{aligned}\right\} \tag{5.6}$$

where we have replaced x_1 derivatives by t_1 derivatives.

We now consider the slow motion in the near field and carry out the procedures of matching for small- and large-amplitude slow motions separately.

6. Slow drift of small-displacement: slow motion in the near field and matching with the far field

6.1. *Large gap* $H = h-D = O(h)$

A geometry most common in practice is one in which the gap between the body and the bottom is not small compared with the water depth.

In the near field the long-period potential is

$$\psi = \epsilon\psi_{10}(x, z, t_1)+\epsilon^2\psi_{20}(x, z, t_1)+O(\epsilon^3), \tag{6.1}$$

up to $O(\epsilon^2)$. While only ψ_{10} is of importance through ψ_{10t_1} to the dynamic pressure at $O(\epsilon^2)$, and hence to the drift force on the body, ψ_{20} is important to the kinematic condition on the body. Both ψ_{10} and ψ_{20} satisfy

$$\psi_{j0xx}+\psi_{j0zz} = 0, \tag{6.2}$$

$$\left.\begin{aligned}\psi_{j0z} = 0, \quad z &= -D, \quad |x| < B,\\ z &= -h, \quad |x| < \infty,\end{aligned}\right\} \tag{6.3}$$

with $j = 1, 2$. On the free surface $z = 0$, $|x| > B$, we have

$$\psi_{10z} = 0, \tag{6.4}$$

$$\psi_{20z} = -\frac{1}{g}(\mathrm{i}\omega\psi_{11}\psi_{11x}^{*}+*)_x \equiv h\frac{\partial U}{\partial x}. \tag{6.5}$$

It can be shown that the velocity U, defined by the parenthesis on the right, approaches the Stokes' drift $U(\pm\infty, t_1)$ at infinity, and its x-derivative vanishes.

Expanding the kinematic boundary condition (3.1) about the mean position $S_0^{\pm}$, we get

$$X_t = \psi_x(S^{\pm}) = \psi_x(S_0^{\pm})+X\psi_{xx}(S_0^{\pm})+O(\epsilon^3). \tag{6.6}$$

At the first order $O(\epsilon)$ we get the zeroth harmonic:

$$\psi_{10x} = 0 \quad \text{on } S_0^{\pm}, \tag{6.7}$$

while at the second order, the zeroth harmonic is

$$\psi_{20x} = X_{10t_1}-(X_{11}\psi_{11xx}^{*}+*) \quad \text{on } S_0^{\pm}. \tag{6.8}$$

In view of (6.2)–(6.5), ψ_{10} is a function only of t_1:

$$\psi_{10} = \psi_{10}(t_1). \tag{6.9}$$

Physically this result is expected on the ground of mass conservation, since the flux through the gap, which is $O(H\epsilon\psi_{10x})$, must be matched to the flux in the far field, i.e.

$$\lim_{|kx|\gg 1} H\frac{\mathrm{d}}{\mathrm{d}x}\epsilon\psi_{10} = \lim_{|kx_1\ll 1|} h\epsilon^2\phi_{10x_1} = O(\epsilon^2). \tag{6.10}$$

This in turn implies that the spatial variation of the slow potential in the near field with respect to the short scale is in general second order in ϵ. At the first order there is only a slow and uniform rise of pressure but no flow.

To find the second-order dynamic boundary condition from (3.3), we divide $S^\pm$ into two intervals $(-h, 0)$ and $(0, \zeta_{11})$. Taking account of the following approximations:

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\epsilon X_{10} = \epsilon^3 X_{10t_1t_1} = O(\epsilon^3),$$

$$\psi_t(S^\pm) = \psi_t(S_0^\pm) + \psi_{tx}X + O(\epsilon^3),$$

$$\int_0^\zeta \psi_t\,\mathrm{d}z = \zeta\psi_t(z=0) + O(\epsilon^3),$$

the second-order, zeroth harmonic of (3.3) is found to be

$$K_1X_{10} = -\rho\left[\int_{S_0^-} P'_{20}\,\mathrm{d}z - \int_{S_0^+} P'_{20}\,\mathrm{d}z + \delta(\mathrm{i}\omega\psi_{11}\zeta_{11}^* + *) + \tfrac{1}{2}\delta(g\zeta_{11}^*\zeta_{11} + *)\right], \tag{6.11}$$

where $K \equiv \epsilon K_1$,

$$P'_{20} = \psi_{10t_1} + |\nabla\psi_{11}|^2 + (\mathrm{i}\omega\psi_{11x}^* X_{11} + *) + gz \tag{6.12}$$

from the Bernoulli equation (2.2), and

$$\delta G \equiv G(-B, 0) - G(B, 0) \tag{6.13}$$

denotes the jump from one side of the body to the other.

With (6.12) incorporated, the right-hand side of (6.11) can be split into contributions by ψ_{10} and ψ_{11}. Since ψ_{11} and ζ_{11} are formally the same as their counterparts for a sinusoidal wave train, the terms involving them must give the usual steady drift force which has been obtained before by Maruo (1960), Newman (1967) and Longuet-Higgins (1977) (see Agnon 1986 for detailed verification). We only give the result here:

$$\rho g|A|^2|R|^2\frac{C_g}{C}, \tag{6.14}$$

where $C \equiv \omega/k$ and A is the amplitude of the free-surface displacement. Equation (6.11) can now be written as

$$K_1X_{10} = -\rho\left[\int_{S_0^-}\psi_{10}\,\mathrm{d}z - \int_{S_0^+}\psi_{10}\,\mathrm{d}z\right]_{t_1} + \rho g|A^2|R|^2\frac{C_g}{C}. \tag{6.15}$$

In view of (6.9), the two integrals above cancel, and the dynamic boundary condition reduces to

$$K_1X_{10} = \rho g|A|^2|R|^2\frac{C_g}{C}. \tag{6.16}$$

Thus our arguments via the multiple-scale expansions reaffirm Newman's result that the slow drift force is approximated by the steady drift-force formula, for a large gap.

We point out that, in the present case, body inertia and radiation damping are negligible at $O(\epsilon^2)$. However, both of these are often included in existing literature in an *ad hoc* manner.

The present theory allows us, in addition, to find the long-wave field, which has not been dealt with in the literature. To determine the wave potentials ϕ_{10}^+ and ϕ_{10}^- we match the slow potential and its gradient on both sides of the near field with the corresponding quantities in the far field, yielding the simple result:

$$\phi_{10}(0_-, t_1) = \psi_{10}(t_1) = \phi_{10}(0_+, t_1). \tag{6.17}$$

Thus to the far-field observer, the near field shrinks to a line at $x_1 = 0$ across which the potential ϕ_{10} is continuous via ψ_{10}. Combination of (5.4) and (6.17) gives one condition:

$$\phi_{10}^+(t_1) - \phi_{10}^-(t_1) = \phi_{10}^{\mathrm{I}}(t_1) + \phi_{10}^{\mathrm{R}}(t_1) - \phi_{10}^{\mathrm{T}}(t_1), \quad x_1 = 0. \tag{6.18}$$

In order to determine the unknown ϕ_{10}^+ and ϕ_{10}^- we need a further condition on $\partial\phi_{10}^+/\partial x_1$ and $\partial\phi_{10}^-/\partial x_1$. This condition is related to the $O(\epsilon^2)$ mean horizontal flux. Viewed in the near field, the second-order mean horizontal flux across a station S_∞^+, where $kx \gg 1$ and $x_1 \ll 1$, is

$$\overline{\int_{-h}^{\zeta} \mathrm{d}z \frac{\partial \psi}{\partial x}\bigg|_{S_\infty^+}} = \zeta_{11} \frac{\partial \psi_{11}^*}{\partial x}\bigg|_{\substack{z=0\\ S_\infty^+}} + * + \int_{-h}^{0} \mathrm{d}z \frac{\partial \psi_{20}}{\partial x}\bigg|_{S_\infty^+}. \tag{6.19}$$

This should be matched to the far-field mean flux:

$$\overline{\int_{-h}^{\zeta} \mathrm{d}z \frac{\partial \phi}{\partial x}\bigg|_{x_1=0^+}} = h\frac{\partial \phi_{10}}{\partial x_1}\bigg|_{x_1=0^+} + \zeta_{11} \frac{\partial \phi_{11}^*}{\partial x}\bigg|_{\substack{x_1=0^+\\ z=0}} + *. \tag{6.20}$$

The overbar stands for averaging over a short-wave period $2\pi/\omega$ and a short wavelength $2\pi/k$. Because ζ_{11} and ϕ_{11} are continuous when passing from the near to the far field, matching of flux implies

$$\frac{\partial \phi_{10}}{\partial x_1}\bigg|_{x_1=0_\pm} = \frac{\partial \psi_{20}}{\partial x}\bigg|_{S_\infty^\pm}. \tag{6.21}$$

Extending the arguments in Agnon & Mei (1985) it is further shown in Appendix A that energy conservation of the first-order waves, $|R|^2 + |T|^2 = 1$, requires also that

$$\frac{\partial \psi_{20}}{\partial x}\bigg|_{S_\infty^+} = U(\infty, t_1) = \frac{\partial \psi_{20}}{\partial x}\bigg|_{S_\infty^-} = U(-\infty, t_1), \tag{6.22}$$

where $U(\pm\infty, t_1)$ is just the flux associated with the Stokes' drift. It then follows that

$$\frac{\partial \phi_{10}}{\partial x_1}\bigg|_{x_1=0_+} = \frac{\partial \phi_{10}}{\partial x_1}\bigg|_{x_1=0_-} = U(\pm\infty, t_1). \tag{6.23}$$

Invoking again energy conservation of short waves we observe from (5.5) that

$$\phi_{10}^{\mathrm{R}} + \phi_{10}^{\mathrm{T}} = \phi_{10}^{\mathrm{I}} \quad \text{at } x_1 = 0. \tag{6.24}$$

Matching the gradients of ϕ_{10} as they appear in (5.6), and using (6.23) and (6.24), we eliminate the locked waves to get

$$\phi_{10}^-(0_-, t_1) = -\phi_{10}^+(0_+, t_1). \tag{6.25}$$

Using (6.24) and (6.25) we write the right-hand side of (6.18) in terms of ϕ_{10}^{R} and the left-hand side in terms of ϕ_{10}^{+}. The result is simply

$$\phi_{10}^{+}(0_+, t_1) = \phi_{10}^{R}(0_-, t_1). \tag{6.26}$$

Thus the free long wave $\phi_{10}^{\pm}$ is completely determined. Its potential amplitude is equal to the long wave locked to the reflected wave group but the speed is different. Substituting (6.25) and (6.26) into (6.17) and (6.18) we determine the near field to leading order:

$$\psi_{10}(t_1) = \phi_{10}(0_\pm, t_1) = \phi_{10}^{I}(0_\pm, t_1). \tag{6.27}$$

Thus ϕ_{10} at $x_1 = 0$ is the same as if the body were absent! This is similar to the Froude–Krylov approximation in the linearized theory of long waves past a small body.

To recapitulate, we first calculate R and T by existing methods in the linearized theory of water-wave diffraction. The steady drift displacement is then given by (6.16), the locked long waves by (5.5) and the free long waves by (6.25) and (6.26). Thus our earlier theory for a sliding block is extended. Since the qualitative features of the results for the long waves are very similar to those for a sliding block already discussed in Agnon & Mei (1985), no numerical results for the former are presented here.

6.2. *Narrow gap* $H = h-D \ll h$

When a ship is moored in very shallow water, the gap between the body and the seabed can be narrow; the analysis of §6.1 does not apply. Consider specifically

$$H \equiv h-D = O(\epsilon h). \tag{6.28}$$

We first show for a vessel with a beam $B/h = O(1)$ that the fast heave and roll amplitudes Z and θ are hydrodynamically restricted to $O(\epsilon^2)$ and can be neglected. Let us denote the fast heave and roll potentials in the gap by $\psi^{(Z)}$ and $\psi^{(\theta)}$, and the related added mass and added moment of inertia by μ_Z and I_θ, respectively. For brevity the subscripts $(\ )_{11}$ are omitted here. The equations of mass conservation in the gap for each case are

$$-H\psi_{xx}^{(Z)} + (-\mathrm{i}\omega)Z = 0, \tag{6.29}$$

$$-H\psi_{xx}^{(\theta)} + (-\mathrm{i}\omega)\theta x = 0. \tag{6.30}$$

Hence, by integration,

$$\psi^{(Z)} = -\mathrm{i}\omega\frac{Z}{H}\tfrac{1}{2}x^2, \tag{6.31}$$

$$\psi^{(\theta)} = -\mathrm{i}\omega\frac{\theta}{H}\tfrac{1}{6}x^3. \tag{6.32}$$

The added mass for heave and the moment of inertia for roll are then

$$\mu_Z = -\frac{\rho}{\omega^2 Z}\int_{-B}^{B}\psi_t^{(Z)}\,\mathrm{d}x = \rho\frac{B^3}{3H} = O(\rho B^3\epsilon^{-1}), \tag{6.33}$$

$$I_\theta = -\frac{\rho}{\omega^2\theta}\int_{-B}^{B}\psi_t^{(\theta)}x\,\mathrm{d}x = \rho\frac{B^5}{15H} = O(\rho B^5\epsilon^{-1}). \tag{6.34}$$

Since $\omega = O(1)$, and the wave pressure is $O(\epsilon)$, the fast heave and roll displacements are $O(\epsilon/\epsilon^{-1}) = O(\epsilon^2)$, and too small to be interesting.

Because of the narrow gap under the body, the scale of z is no longer h. Previous conclusions on ψ_{10} no longer hold since strong blockage occurs. The slow current under the body becomes large ($O(\epsilon)$) and there is a large pressure gradient or difference between the two sides of the body. Apart from this difference, the flow in the gap has only an $O(\epsilon^2)$-effect on mass flux, which does not play a significant role in ψ_{10} outside the gap on either side of the body.

We now introduce $\epsilon\psi'_{10}$ and $\epsilon^2\psi'_{20}$ so that

$$\epsilon\psi'_{10}+\epsilon^2\psi'_{20} = \epsilon\psi_{10}+\epsilon^2\psi_{20} \tag{6.35}$$

with ψ'_{20} satisfying (6.5), and

$$\psi'_{20x} = -(X_{11}\psi^*_{11xx}+*)+U(\pm\infty,t_1) \quad \text{on } S_0^{\pm} \tag{6.36}$$

on the sidewalls. In view of the vanishing of $\partial U/\partial x$ at $x\rightarrow\infty$ we must have

$$\psi'_{20}\rightarrow 0, \quad |x|\rightarrow\infty. \tag{6.37}$$

It follows from (6.6), or (6.7) and (6.8), that

$$\psi'_{10x} = X_{10t_1}-U(\pm\infty,t_1) \quad \text{on } S_0^{\pm}. \tag{6.38}$$

Otherwise ψ'_{10} satisfies the same conditions as ψ_{10}, and ψ'_{20} as ψ_{20}, i.e. (6.2)–(6.4). ψ'_{10} now contains both first- and second-order effects. Clearly $\epsilon^2\psi'_{20}$ corresponds to a radiation problem which will give rise to $O(\epsilon^3)$ long-period pressure only.

Note that $\epsilon\psi'_{10}$ corresponds to the flow in a channel with a rigid top and bottom, induced by a block moving longitudinally at the velocity $\epsilon^2(X_{10t_1}-U)$. This is a typical problem in the linearized theory of long-wave scattering by a body. Since the long wave gives rise to an ambient current $\epsilon^2 v$, the velocity of the current relative to the body is $\epsilon^2(v-X_{10t_1}-U)$. It is known (Flagg & Newman 1971) for a *fixed* body in long waves that the outer approximation of the near-field potential is given by

$$\epsilon Q(t_1)+\epsilon^2(v-X_{10t_1}+U)\,x\pm\epsilon^2(v-X_{10t_1}+U)\,c, \quad x\rightarrow\pm\infty, \tag{6.39}$$

where c is the blockage coefficient defined here to have the dimension of length. The quantities Q and V must be determined by matching with the far field. Since the body is *moving* in the stationary frame of reference, we superimpose a countercurrent $\epsilon^2(X_{10t_1}-U)$ and get

$$\epsilon\psi'_{10}\sim\epsilon Q(t_1)+\epsilon^2 vx\pm\epsilon^2(v-X_{10t_1}+U)\,c, \quad x\rightarrow\pm\infty. \tag{6.40}$$

For a narrow gap the blockage coefficient c is known to be a large quantity,

$$c = \frac{Bh}{H}+O\,(\log\epsilon) = O(\epsilon^{-1}) \tag{6.41}$$

(Flagg & Newman 1971). Hence the variation of ψ'_{10} across the body is $O(1)$. This implies that the slow potential must contribute a net force on the body according to (6.15), unlike the usual cases of $H/h = O(1)$.

We now match the outer expansion of the near field, (6.40), to the inner expansion of the far field, (5.6), giving

$$\left.\begin{aligned}
Q(t_1)+[v-X_{10t_1}+U]\,\epsilon c &= [\phi^{\mathrm{T}}_{10}+\phi^{+}_{10}]_{x_1=0},\\
Q(t_1)-[v-X_{10t_1}+U]\,\epsilon c &= [\phi^{\mathrm{I}}_{10}+\phi^{\mathrm{R}}_{10}+\phi^{-}_{10}]_{x_1=0},\\
v &= \left[-\frac{\phi^{\mathrm{I}}_{10}}{C_g}+\frac{\phi^{\mathrm{R}}_{10}}{C_g}+\frac{\phi^{-}_{10}}{(gh)^{\frac{1}{2}}}\right]_{t_1},\\
v &= \left[-\frac{\phi^{\mathrm{T}}_{10}}{C_g}-\frac{\phi^{+}_{10}}{(gh)^{\frac{1}{2}}}\right]_{t_1}.
\end{aligned}\right\} \tag{6.42}$$

Integrating the last two equations above with respect to t_1, we get

$$\phi_{10}^{+} = -\phi_{10}^{-}, \quad x_1 = 0. \tag{6.43}$$

Using $U(+\infty, t_1) = U(-\infty, t_1)$ and eliminating Q, we find the potential jump across the body,

$$\begin{aligned}[\phi_{10}]_{x_1=0_+} - [\phi_{10}]_{x_1=0_-} &= \phi_{10}^{T} + \phi_{10}^{+} - (\phi_{10}^{I} + \phi_{10}^{R} + \phi_{10}^{-}) \\ &= 2c_1 \left\{ \left[-\frac{\phi_{10}^{T}}{C_g} - \frac{\phi_{10}^{+}}{(gh)^{\frac{1}{2}}} \right]_{t_1} - X_{10t_1} + U \right\},\end{aligned} \tag{6.44}$$

where $c_1 \equiv \epsilon c = O(1)$. This is also the slow-potential difference $[\psi'_{10}]_{S_\infty^-} - [\psi'_{10}]_{S_\infty^+}$ between the two outer limits of the near fields, i.e. between S_∞^+ and S_∞^-. Because the gap is small, the slow potential is different from ψ'_{10} at $S_\infty^\pm$ only in the $O(\epsilon)$ neighbourhood of the gap entrance beneath $S_0^\pm$. Hence to leading order (6.44) gives the net long-period pressure on the body. Equation (6.15) becomes

$$K_1 X_{10} = \rho h[\phi_{10}^{+} + \phi_{10}^{T} - (\phi_{10}^{I} + \phi_{10}^{R} + \phi_{10}^{-})]_{t_1} + \rho g |A|^2 |R|^2 \frac{C_g}{C}, \quad x_1 = 0. \tag{6.45}$$

Because the gap is narrow, the reflection coefficient R is the same as that for a sliding block (cf. end of §4.1) computed in Agnon & Mei (1985).

We shall now eliminate $\phi_{10}^{\pm}$ between (6.43) and (6.44). From (6.24), (6.43) and (6.45) we get

$$\pm 2\phi_{10t_1}^{\pm} = \frac{K_1 X_{10}}{\rho h} + 2\phi_{10t_1}^{R} - \frac{g}{h}|A^2|R|^2 \frac{C_g}{C}. \tag{6.46}$$

Differentiating (6.44) with respect to t_1 we can substitute its left-hand side into the right-hand side of (6.45). Using (6.46) for $\phi_{10t_1}^{+}$, we get finally

$$\begin{aligned}X_{10t_1t_1} &+ \frac{K_1}{2\rho h(gh)^{\frac{1}{2}}} X_{10t_1} + \frac{K_1}{2\rho h c_1} X_{10} \\ &= \left[-\left(\frac{C_g}{(gh)^{\frac{1}{2}}} |R|^2 + 1 - |R|^2 \right) \frac{C_g}{gh - C_g^2} \frac{g}{2h} S \frac{\partial}{\partial t_1} + \frac{C_g}{C} |R|^2 \frac{g}{2hc_1} \right] |A|^2,\end{aligned} \tag{6.47}$$

where

$$S \equiv \frac{2C_g}{C} - \tfrac{1}{2} \tag{6.48}$$

is the well-known factor appearing in the radiation stress. Details are given in Agnon (1986). Equation (6.47) is similar to that for a damped oscillator (note that the blockage coefficient c plays the role of an effective mass). Once it is solved for certain initial conditions, we get from (6.46) $\phi_{10}^{\pm}(\pm 0, t_1)$, which then gives $\phi_{10}^{\pm}(t_1 \mp (gh)^{\frac{1}{2}} x_1)$. Through the Bernoulli equation, the free-surface elevation associated with the free long waves is readily obtained from

$$\zeta_{20}^{\pm} = -\frac{1}{g} \phi_{10t_1}^{\pm}. \tag{6.49}$$

In the limit as $H \rightarrow 0$, there is no gap and the blockage coefficient $c_1 \rightarrow \infty$; the last term on each side of (6.47) vanishes and the resulting equation may be integrated with respect to t_1, yielding

$$X_{10t_1} + \frac{K_1}{2\rho h(gh)^{\frac{1}{2}}} X_{10} = -\left(\frac{C_g}{(gh)^{\frac{1}{2}}} |R^2| + 1 - |R|^2 \right) \frac{C_g}{gh - C_g^2} \frac{g}{2h} S|A|^2, \tag{6.50}$$

which is identical to the equation for a sliding block (Agnon & Mei 1985). On the other hand, as the gap becomes large, $H = O(1)$, $c_1 = O(\epsilon)$ and the last term on each side of (6.47) becomes the dominant term, and (6.16) is recovered. Therefore, (6.47) may be regarded as practically valid for all gap widths although it is established for a narrow gap only!

Note from (6.47) that the normalized response is independent of the value of ϵ.

7. Slow drift of small displacement: Examples for special incident envelopes for the narrow-gap case

We shall now give explicit solutions to (6.47) for the following wave envelopes: (*a*) a steady sinusoidal envelope; (*b*) the sudden start of a sinusoidal envelope; (*c*) the gradual start of a sinusoidal wave; and (*d*) a wave packet.

(*a*) *A quasi-steady sinusoidal envelope*

Let the incident wave have the scaled amplitude

$$A = A_0 \sin \Omega t_1 \quad \text{at } x_1 = 0 \quad \text{for all } t_1, \tag{7.1}$$

so that the actual amplitude is ϵA_0. The solution to (6.47) is simply

$$X_{10} = \mathrm{Re}\,[X'_{10}\,\mathrm{e}^{-2\mathrm{i}\Omega t_1} + X''_{10}]. \tag{7.2}$$

The amplitude of the oscillatory component is

$$X'_{10} \equiv -\tfrac{1}{2}D\left[-4\Omega^2 - \frac{\mathrm{i}\Omega K_1}{\rho h (gh)^{\frac{1}{2}}} + \frac{K_1}{2\rho h c_1}\right]^{-1}, \tag{7.3}$$

with $$D \equiv \left\{2\mathrm{i}\Omega\left(\frac{C_g}{(gh)^{\frac{1}{2}}}|R|^2 + 1 - |R|^2\right)\frac{C_g}{gh - C_g^2}\frac{g}{2h}S + \frac{C_g}{C}\frac{g}{2hc_1}|R|^2\right\}A_0^2. \tag{7.4}$$

The steady-state component is

$$X''_{10} \equiv \frac{1}{2}\frac{g\rho}{K_1}\frac{C_g}{C}A_0^2|R|^2. \tag{7.5}$$

(*b*) *Sudden start of a sinusoidal envelope*

Let $$A = H(t_1)\,A_0 \sin \Omega t_1, \tag{7.6}$$

where H is the Heaviside step function. The solution of the homogeneous part of (6.47) are

$$X_{10} \propto \mathrm{e}^{\alpha_1 t_1},\ \mathrm{e}^{\alpha_2 t_1}, \tag{7.7}$$

with $$\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix} = \frac{-K_1}{4\rho h (gh)^{\frac{1}{2}}} \pm \left[\frac{K_1^2}{16\rho^2 h^2 gh} - \frac{K_1}{2\rho h c_1}\right]^{\frac{1}{2}}. \tag{7.8}$$

It is assumed that the square root above does not vanish so that $\alpha_1 \neq \alpha_2$. Otherwise the solutions are $\mathrm{e}^{\alpha_1 t_1}$ and $t_1\,\mathrm{e}^{\alpha_1 t_1}$; the results are not qualitatively more interesting.

With the initial conditions $X_{10}(0) = X_{10t_1}(0) = 0$ we get the transient sway:

$$X_{10} = \mathrm{Re}\left\{X''_{10}\left[1 - \frac{\alpha_2\,\mathrm{e}^{\alpha_1 t_1} - \alpha_1\,\mathrm{e}^{\alpha_2 t_1}}{\alpha_2 - \alpha_1}\right] + X'_{10}\left[\mathrm{e}^{-2\mathrm{i}\Omega t_1} - \frac{(\alpha_2 + 2\mathrm{i}\Omega)\,\mathrm{e}^{\alpha_1 t_1} - (\alpha_1 + 2\mathrm{i}\Omega)\,\mathrm{e}^{\alpha_2 t_1}}{\alpha_2 - \alpha_1}\right]\right\}, \tag{7.9}$$

where X'_{10} and X''_{10} are defined in (7.3) and (7.5).

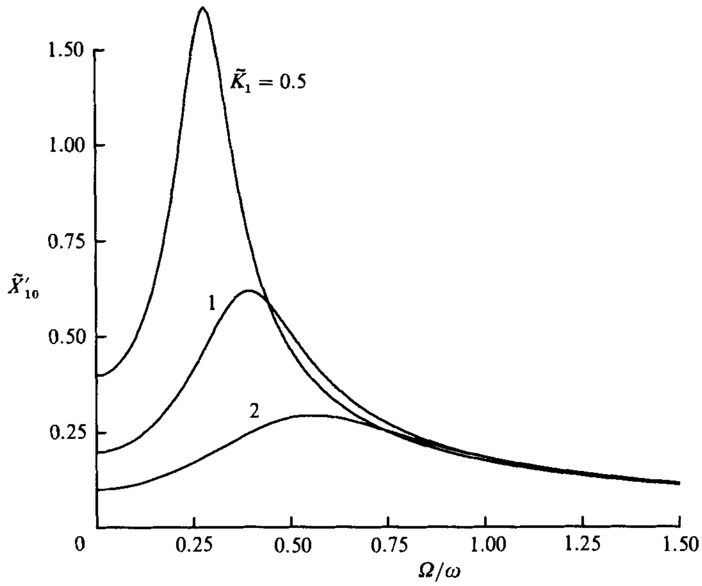

FIGURE 2. Normalized amplitudes of the slow-sway displacement $\tilde{X}'_{10} = h|X'_{10}|/A_0^2$ for Case (a), a sinusoidal envelope; $\tilde{K}_1 = K/\epsilon\rho gh = K_1/\rho gh = 0.5$, 1 and 2; $kh = 1.0$; $c_1/h = 1$ (narrow gap).

(c) Gradual start of a uniform wave train

Let the envelope begin from zero at $t_1 = 0$, grow sinusoidally to $\pi/2\Omega$, and then be kept uniform:

$$A = A_0\left[H(t_1)H\left(\frac{\pi}{2\Omega}-t_1\right)\sin\Omega t_1 + H\left(t_1-\frac{\pi}{2\Omega}\right)\right]. \tag{7.10}$$

Then X_{10} is given by (7.9) for $0 \leqslant t_1 \leqslant \pi/2\Omega$, and

$$X_{10} = \mathrm{Re}\left\{2X''_{10}\left[1-\frac{\alpha_2\,\mathrm{e}^{\alpha_1(t_1-\pi/2\Omega)}-\alpha_1\,\mathrm{e}^{\alpha_2(t_1-\pi/2\Omega)}}{\alpha_2-\alpha_1}\right]\right.$$
$$+\left[\alpha_2 X_{10}\left(\frac{\pi}{2\Omega}\right)-X_{10t_1}\left(\frac{\pi}{2\Omega}\right)\right]\frac{\mathrm{e}^{\alpha_1(t_1-\pi/2\Omega)}}{\alpha_2-\alpha_1}$$
$$\left.-\left[\alpha_1 X_{10}\left(\frac{\pi}{2\Omega}\right)-X_{10t_1}\left(\frac{\pi}{2\Omega}\right)\right]\frac{\mathrm{e}^{\alpha_2(t_1-\pi/2\Omega)}}{\alpha_2-\alpha_1}\right\}, \quad t_1 > \frac{\pi}{2\Omega}. \tag{7.11}$$

As $t_1 \uparrow \infty$, X_{10} tends to $2X''_{10}$, which is the steady drift displacement, and is also the value given by (6.16).

(d) A wave packet

Let the envelope have a finite duration from $t_1 = 0$ to $t_1 = \pi/\Omega$:

$$A = H(t_1)H\left(\frac{\pi}{\Omega}-t_1\right)A_0\sin\Omega t_1, \tag{7.12}$$

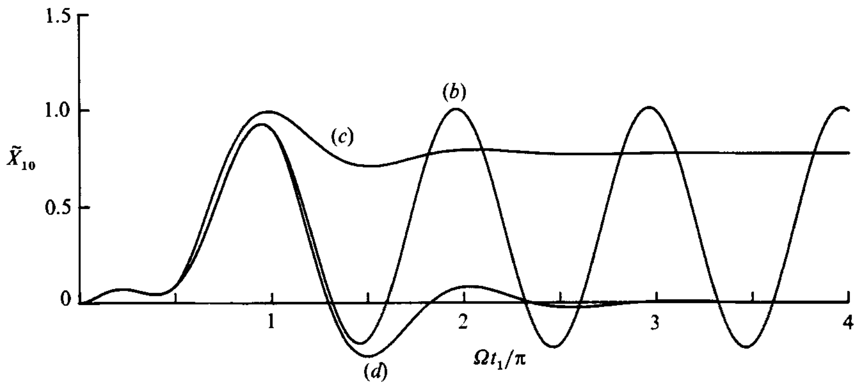

FIGURE 3. Transient slow displacement for a block with a narrow gap. Case (*b*), a sinusoidal envelope starting from rest; Case (*c*), uniform envelope starting from rest; Case (*d*), a pulse envelope. $c_1/h = 1$; $kh = 1$; $\Omega/\omega = 0.4$; $K_1/\rho g h = K/\epsilon\rho g h = 1$.

then X_{10} is given by (7.9) for $0 \leqslant t_1 \leqslant \pi/\Omega$, and

$$X_{10} = \operatorname{Re}\left\{\left[\alpha_2 X_{10}\left(\frac{\pi}{\Omega}\right) - X_{10t_1}\left(\frac{\pi}{\Omega}\right)\right]\frac{e^{\alpha_1(t_1-\pi/\Omega)}}{\alpha_2-\alpha_1} - \left[\alpha_1 X_{10}\left(\frac{\pi}{\Omega}\right) - X_{10t_1}\left(\frac{\pi}{\Omega}\right)\right]\frac{e^{\alpha_2(t_1-\pi/\Omega)}}{\alpha_2-\alpha_1}\right\}, \quad \frac{\pi}{\Omega} \leqslant t_1. \quad (7.13)$$

We now present numerical results in dimensionless parameters denoted by tildes. In all the computations presented in this section we chose $kh = 1$, $\tilde{M} \equiv M/\rho h^2 = 1$ and $\tilde{c}_1 \equiv c_1/h = 1$. The normalized mooring constant is $\tilde{K}_1 \equiv K_1/\rho g h = K/(\epsilon \rho g h)$.

In figure 2, the normalized slow-sway amplitude

$$\tilde{X}'_{10} = \frac{h|X'_{10}|}{A_0^2} = \frac{\epsilon h|\hat{X}'_{10}|}{\hat{A}_0^2},$$

where a circumflex denotes physical quantities unscaled by ϵ, is plotted versus $\tilde{\Omega} \equiv \Omega/\omega = \hat{\Omega}/\epsilon\omega$, the wave modulation ratio, for three values of $\tilde{K}_1$. The resonance peak occurs near $2\Omega/\omega = (K_1/2\rho h c_1)^{\frac{1}{2}}$. Thus the narrower the gap, the greater the blockage and the sharper the resonance.

To help gain some idea of the physical magnitude involved, we may take the following sample values: depth $h = 10$ m, vessel width $2B = 10$ m, wavelength $\lambda = 63$ m (or $k = 0.1$/m) which corresponds to a wave period of 7.2 s. Corresponding to the dimensionless parameters of c_1 and $\tilde{K}_1$ chosen, the gap width is 5ϵ (thus $H = 0.5\ m$ if ϵ is 0.1), and the spring constant is 10^4 N/m if $\tilde{K}_1 = 1$. Each unit of the abscissa (Ω/ω) corresponds to a 72 s period of the modulation envelope. If the incident wave amplitude is 3 m each unit of the ordinate $\tilde{X}_{10}$ stands for a slow-sway amplitude of ~ 1 m.

In figure 3, the slow-drift displacement $\tilde{X}_{10}$ caused by transient wave envelopes is shown according to (7.3), (7.11), and (7.13) versus the non-dimensional slow time, Ωt_1. For all inputs, the modulational frequency Ω is chosen to be near the resonant peak with $\Omega/\omega = 0.4$ (cf. figure 2). For Case (*b*) the oscillatory part of the drift motion is larger than the mean. For Case (*c*) there can be some initial overshoot, after which the final steady state is quickly reached. For a pulse envelope the negative overshoot is followed by quick attenuation.

8. Slow sway of large amplitude: fast motion in the near and far fields

In many practical situations, where the mooring is very weak the slow-sway displacement can be much larger than the wave amplitude. It is no longer feasible to carry out the Taylor expansion about the rest position of the body: modifications are required.

Specifically, we assume

$$K = \epsilon^2 K_2 = O(\epsilon^2) \tag{8.1}$$

and $H = O(h)$ and the total displacement can be large:

$$X = O(1). \tag{8.2}$$

Instead of (4.1) we let

$$X = \sum_{n=0}^{\infty} \epsilon^n \sum_{m=-n}^{n} X_{nm} \,\mathrm{e}^{-\mathrm{i}m\omega t}, \tag{8.3}$$

where the series begins at $n = 0$ and $X_0 \equiv X_{00}$ is the slow-sway displacement of order unity, i.e. comparable with h or k^{-1}. The corresponding drift velocity is first order,

$$\frac{\partial X_0}{\partial t} = \epsilon X_{0t_1} = O(\epsilon). \tag{8.4}$$

We define a moving coordinate system that follows the slow motion X_0, in order to study the fast potential ψ_{11}. Let

$$(x', z', t') \equiv (x - X_0, z, t) \tag{8.5}$$

be the moving coordinates in terms of which the velocity potential is

$$\Phi'(x', z', t') \equiv \Phi(x, z, t). \tag{8.6}$$

The spatial gradient does not change under the transformation (4.12) so that

$$\nabla' \Phi' = \nabla \Phi, \tag{8.7}$$

but the time derivative changes in accordance with

$$\Phi'_{t'} = \Phi_t - \epsilon X_{0t_1} \Phi_x. \tag{8.8}$$

Let us denote the near-field potential by ψ'

$$\Phi' = \psi'; \quad x', z' = O(1) \tag{8.9}$$

and expand it into orders and harmonics as in (2.7)

$$\psi' = \sum_{n=1}^{\infty} \epsilon^n \sum_{m=-n}^{n} \psi'_{nm} \,\mathrm{e}^{-\mathrm{i}m\omega t'}, \tag{8.10}$$

The first-order fast potential ψ_{11} satisfies formally (4.2)–(4.5) when all the unprimed variables are replaced by the primed variables. Therefore, the solution in the primed coordinates follows trivially from that in the unprimed coordinates, and can be regarded as known.

9. Slow sway of large amplitude: slow motion in the near and far fields

From (8.8), the slow-drift velocity affects the short-period pressure at $O(\epsilon^2)$ but the long-period pressure at $O(\epsilon^3)$. It follows that the drift force on a body is still given by the conventional theory with the amplitude varying slowly in time. If desired, the correction can in principle be computed in a manner similar to the theory of a ship cruising in waves (see e.g. Grue & Palm 1985 and Huijsmans & Hermans 1985). Nevertheless, the leading-order correction to the drift force must be proportional to the small 'current speed'. Without calculating the details we shall formally express the 'current effect' on the drift force by

$$\rho g \lambda' \epsilon^2 |A|^2 \epsilon X_{0t_1} = O(\epsilon^3)\lambda', \tag{9.1}$$

with $\lambda' = O(1)$.

To the first order $O(\epsilon)$, the slow kinematic boundary condition on the body can be approximated from (6.6)

$$\psi_{10_x} = X_{0t_1} \quad \text{on } S_0^\pm, \quad x = \pm B + X_0(t_1), \; -D < z < 0. \tag{9.2}$$

The dynamic boundary condition on the body is obtained in the same way as (6.15) for the small-amplitude slow sway, except that the body inertia, which is now of order ϵ^2, must now be added. A consistent assumption for the mooring force is $K = \epsilon^2 K_2$, i.e. very weak. We now have

$$MX_{0t_1t_1} + K_2 X_0 = -\rho\left[\int_{S^-} \psi_{10}\,\mathrm{d}z - \int_{S^+} \psi_{10}\,\mathrm{d}z\right]_{t_1} + \rho g |A|^2 |R|^2 \frac{C_g}{C}. \tag{9.3}$$

The slow potential ψ_{10} can be separated into two parts: the first part $\psi_{10}^{(1)}$ is a function of t_1 only and is the near field of the slow potential given by (6.24)–(6.27). Its corresponding far field $\phi_{10}^{(1)}$ consists of the locked long waves which travel with the short-wave groups (I, R and T) and of the free wave $\phi_{10}^\pm$. In particular, $\psi_{10}^{(1)}$ satisfies

$$\psi_{10x}^{(1)} = 0 \quad \text{on } S_0^\pm. \tag{9.4}$$

Because of this homogeneous condition, $\psi_{10}^{(1)}$ may be called the diffracted free wave. The remaining part of the slow potential, $\psi_{10}^{(2)}$, is related to the radiated wave, satisfying (9.2), the inhomogeneous condition and the radiation conditions that the long waves are outgoing from the body. $\psi_{10}^{(2)}$ is simply the solution to the linear problem for wave radiation by the slow-sway motion. The determination of $\psi_{10}^{(2)}$ and the corresponding ϕ_{10} is an exercise in matched asymptotics similar to Beck & Tuck (1972). For convenience, the essential results are cited in Appendix B. In particular we find the far fields to be

$$\phi_{10}^{(2)} = \pm\epsilon\,\mathrm{Re}\{X_0 \alpha \exp[-2\mathrm{i}\Omega(t_1 \mp x_1/(gh^{\frac{1}{2}})]\} \quad x_1 \gtrless 0, \tag{9.5}$$

where

$$\alpha = -\frac{2\mathrm{i}\Omega c}{\epsilon \mathrm{i} k_1 c - 1}, \tag{9.6}$$

and c is the same blockage coefficient as in (6.41), and $k_1 = 2\Omega/(gh)^{\frac{1}{2}} = O(1)$ is the normalized long wavenumber. It may be pointed out that matching ensures the continuity of pressure and mass flux.

The associate drift force due to $\psi_{10}^{(2)}$ is

$$\begin{aligned}\epsilon^2 F_{20}^{(2)} &= -\epsilon^2 \rho\left[\int_{S^-} \psi_{10t_1}^{(2)}\,\mathrm{d}z - \int_{S^+} \psi_{10t_1}^{(2)}\,\mathrm{d}z\right] \\ &= \epsilon^2 \rho\,\mathrm{Re}[-2\mathrm{i}\Omega X_0 \mathrm{e}^{-2\mathrm{i}\Omega t_1}(2h\alpha - 2\mathrm{i}\Omega 2BD)].\end{aligned} \tag{9.7}$$

The result has been derived by Beck & Tuck (1972, equation 5.8) who studied a slender body in shallow water. We can define the added-mass and damping coefficients μ and λ by

$$-F_{20}^{(2)} = (-4\Omega^2\mu - 2\mathrm{i}\Omega\lambda) X_0 \mathrm{e}^{-2\mathrm{i}\Omega t_1}$$

$$= 4\Omega^2 X_0 \left[\frac{2ch}{\epsilon \mathrm{i} k_1 c - 1} + 2BD\right] \mathrm{e}^{-2\mathrm{i}\Omega t_1}, \tag{9.8}$$

i.e.

$$\mu = 2\rho\left[-BD + \frac{ch}{1+(\epsilon k_1 c)^2}\right], \quad \lambda = \frac{4\Omega\rho\epsilon k_1 hc^2}{[1+(\epsilon k_1 c)^2]}. \tag{9.9}$$

We may now rewrite (9.3) as

$$(M+\mu) X_{0t_1t_1} + \lambda X_{0t_1} + K_2 X_0 = \rho g |A|^2 \left(|R|^2 \frac{C_g}{C} + \epsilon\lambda' X_{0t_1}\right). \tag{9.10}$$

Note from (9.10) that for a wide gap $c/h = O(1)$ or $H/h = O(1)$ and $ch = O(BD)$; the apparent mass μ is of the same order as the actual body mass but the radiation damping force associated with λ is much smaller than the rest by the factor $O(\epsilon)$. This suggests the possibility of large resonance if $|A|^2$ has the modulational frequency close to $[K_2/(M+\mu)]^{\frac{1}{2}}$. However, near resonance, where the body inertia and the elastic mooring force nearly cancel each other, real-fluid effects so far ignored are important and serve to make the resonance finite. To the leading order (9.10) is surprisingly simple in that one only needs the apparent mass μ of the body for the long-period oscillations, the drift force being given by the standard formula as stipulated by Newman (1974). To account for this weak radiation damping consistently with the inviscid theory, it is necessary to go to $O(\epsilon^3)$ and further to include the effect of the λ' term. This is not very rewarding in view of the more important effects of viscosity and flow separation. There is however a special circumstance where radiation damping can be more important than the λ' term. Referring to (6.41), the blockage coefficient depends on the ratio B/H, which can be large. Consider $c/h = O(1/\epsilon^{\frac{1}{2}})$, then $\mu = O(1/\epsilon^{\frac{1}{2}})$ and $\lambda = \epsilon^{\frac{1}{2}}$. Let the damping be moderately weak so that $K = (\epsilon^{\frac{3}{2}} K_{\frac{3}{2}})$ or $K_2 = K_{\frac{3}{2}}/\epsilon^{\frac{1}{2}}$. Then the balance of response to forcing in (9.10) implies that $X_0 = O(\epsilon^{\frac{1}{2}})$. In this case the radiation damping term is still small: $\lambda X_{0t_1} = O(\epsilon)$. However the drift force induced by the λ' term should reduce with X_{10t_1} and be $O(\epsilon^{\frac{3}{2}})$, and is still relatively unimportant.

10. Examples for special incident envelopes for large slow sway

For the cases studied in §7 the analytical results in §7 can be carried over directly to the present case, with X_{10} replaced by X_0. The only modifications are that X'_{10} defined by (7.3) must be replaced by

$$X'_0 = -\frac{1}{2}\frac{C_g}{C}\rho g |R|^2 A_0^2 [-(M+\mu) 4\Omega^2 - 2\mathrm{i}\Omega\lambda + K_2]^{-1}, \tag{10.1}$$

and $\alpha_{1,2}$ of (7.8) are now given by

$$\alpha_{1,2} = \frac{-\lambda \pm [\lambda^2 - 4K_2(M+\mu)]^{\frac{1}{2}}}{2(M+\mu)}. \tag{10.2}$$

In general a long floating cylinder has three degrees of freedom, and it is routine to solve numerically the linearized diffraction and radiation problem for ψ_{11} allowing all three modes of motion. We have therefore calculated the reflection and

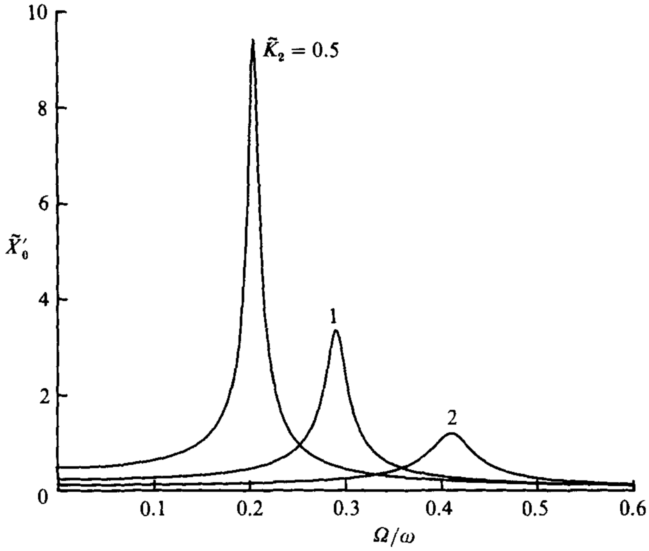

FIGURE 4. Amplitudes of the normalized slow sway $\tilde{X}'_0 = hX_0/A_0^2$ versus the modulation ratio Ω/ω, for $\tilde{K}_2 = K_2/\rho gh = K/\epsilon^2\rho gh = 0.5$, 1 and 2; $c/h = 1$; $kh = 1.6$; $\epsilon = 0.1$.

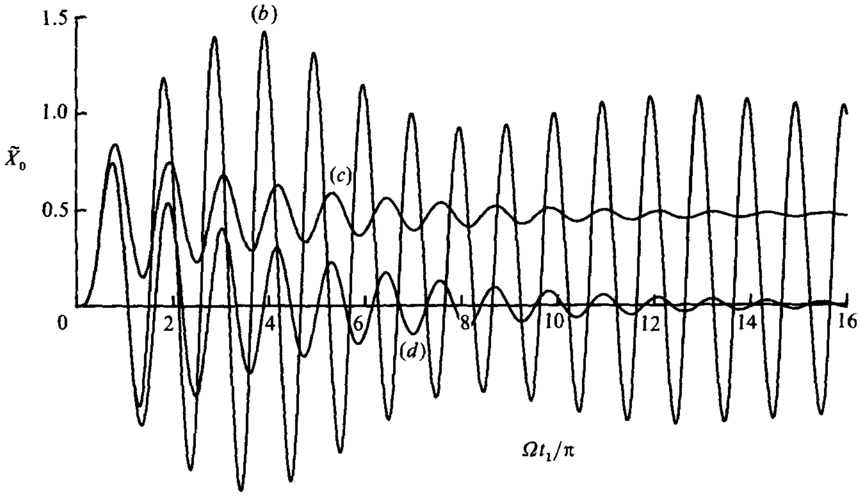

FIGURE 5. Transient slow sway of large amplitude for a floating rectangular cylinder. Case (*b*), a sinusoidal envelope starting from rest; Case (*c*), a uniform envelope starting from rest; Case (*d*), a pulse envelope $\tilde{X}_0 = hX_0/A_0^2$. The parameters are $\Omega/\omega = 0.33$; $\tilde{K}_2 = K_2/\rho gh = 1$; $c/h = 1$; $kh = 1.6$; $\epsilon = 0.1$.

transmission coefficients R and T by allowing all three modes, which are unconstrained by the weak mooring force. Specifically we have carried our calculations for $kh = 1.6$ and a square cylinder with sides $2B$ and draught $D = B$, in water of depth $h = 2B$. The density of the cylinder is homogeneous and is equal to half the water density. After finding the metacentre (at $z = \frac{1}{6}B$) and the moments of inertia, we have computed the reflection coefficient to be $|R^2| = 0.7447$, which includes the effects of diffraction and radiation. The added mass in long waves can be found to be $\mu = 1.5\,\rho h^2$ (Flagg & Newman, 1971). Sedov's formula (Beck & Tuck, 1972) $\mu = 2\rho(hc - BD)$ then implies a blockage coefficient $c/h = 1$. The slow sway of large amplitude is affected by the three first-order modes only indirectly through R and T, the slow heave and roll being $O(\epsilon)$ smaller by comparison, as pointed out near the end of §3.

In figure 4 the value of the normalized slow-sway amplitude for a periodic envelope, $\tilde{X}_0 \equiv hX_0/A_0^2 = \epsilon^2 h\hat{X}_0/\hat{A}_0^2$, is plotted versus the modulation ratio Ω/ω for various choices of the parameters. λ' is omitted in these calculations. It is clear from (9.9) and (9.10) that increasing c as well as increasing K_2 will reduce X_0. Because of the weak radiation damping the resonant peaks are sharp. The very high peak for $\tilde{K}_2 = 0.5$ is of qualitative value only, as flow separation and other higher order effects must be important. In figure 5 transient responses corresponding to the transient inputs of Cases (*b–d*) of §7 are plotted. The long timescale of the transient part is chosen so that the corresponding frequency is not too far from the resonant peak shown in figure 4 for $\tilde{K}_2 = 1$. Again, due to weak radiation damping, there is a long period of reverberation at the natural frequency, after the transient part of the input has gone.

To see these results in physical scales we take a sample cylinder in water of $h = 10$ m depth. For $\epsilon = 0.1$, the corresponding unit slow sway in the graph is approximately 10 m if the incident wave amplitude is 3 m.

11. Conclusions

By a multiple-scale analysis we have been able to separate the low-frequency part of the second-order fluid motion from the high-frequency part. With further use of matched asymptotics, we have shown for a rectangular cylinder that (i) the transient slow-drift problem can be solved analytically without solving explicitly for any second-order potential, and (ii) the slow motion in the near field is accompanied by the propagation of long waves in the far field.

We have also examined the effect of the blockage coefficient, which is a measure of the obstruction by the body to long waves, and the effect of mooring stiffness. Specifically when the mooring is not too weak, the slow sway is comparable in magnitude with fast sway. If, furthermore, the blockage is not large, then the drift response is passive as predicted by Newman (1974). The local slow potential is hardly affected by the presence of the relatively small body. However, if the blockage coefficient is large, added-mass and damping coefficients of the slow motion become significant. The differential equation of slow motion becomes second order and moderate resonance may occur. The slowly varying potential, ignored by most previous authors, is shown to play an important role here. The response to a transient input may exhibit considerable overshooting which should be relevant to a long vessel in a shallow harbour.

We have also extended the analysis of Triantafyllou (1982) for the case of very weak mooring and clarified the conditions under which the slow sway can become $O(1)$. Within the realm of the potential theory, damping due to the radiation of long waves and the change of drift forces due to the 'forward speed' effect of the slow sway on the short waves are both small, and strong resonance can occur. Damping due to real-fluid effects is of course important but much too difficult to be predicted theoretically.

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Appendix A. Integrals concerning $\partial\psi_{20}/\partial x$ at S_∞^+ and S_∞^-

We recall that ψ_{20} satisfies (6.2)–(6.5) and (6.8), thus ψ_{20} accounts for the short-scale variation of the slow potential. Let us apply Gauss' theorem to $\nabla\psi_{20}$ in a control volume bounded by the surfaces $z=-h$ and $z=0$ on the bottom and top, the surfaces S_∞^+ and S_∞^- on the right and left, and the rest position of the body, with the result

$$\left[\int_{S_\infty^+}-\int_{S_\infty^-}+\int_{S_0^+}-\int_{S_0^-}\right]\mathrm{d}z\,\psi_{20x}+\left[\int_{-\infty}^{-B}+\int_{B}^{\infty}\right]\mathrm{d}x\,\psi_{20z}\bigg|_{z=0}$$
$$+\int_{-B}^{B}\mathrm{d}x\,\psi_{20z}\bigg|_{z=-D}-\int_{-\infty}^{\infty}\mathrm{d}x\,\psi_{20z}\bigg|_{z=-h}=0. \quad \text{(A 1)}$$

Now, the integrals along the free surface are

$$\left[\int_{-\infty}^{B}+\int_{B}^{\infty}\right]\psi_{20z}\,\mathrm{d}z=h\left(\int_{-\infty}^{-B}+\int_{B}^{\infty}\right)U_x\,\mathrm{d}x=hU\bigg|_{-\infty}^{-B}+hU\bigg|_{B}^{\infty} \quad \text{(A 2)}$$

after using (6.5). The integrals along the vertical walls of the body are

$$\left(\int_{S_0^+}-\int_{S_0^-}\right)\mathrm{d}z\,\psi_{20x}=-\left\{\int_{S_0^+}-\int_{S_0^-}\right\}[X_{11}\,\psi_{11xx}^*+*]\,\mathrm{d}z+\left(\int_{S_0^+}-\int_{S_0^-}\right)X_{10t_1}\,\mathrm{d}z. \quad \text{(A 3)}$$

Since ψ_{11} satisfies Laplace's equation we have

$$\begin{aligned}\int_{-D}^{0}(X_{11}\,\psi_{11xx}^*+*)\,\mathrm{d}z&=\left[-X_{11}\int_{-D}^{0}\psi_{11zz}^*\,\mathrm{d}z+*\right]\\&=-X_{11}\,\psi_{11z}^*(\pm B,0)+*=-\frac{\mathrm{i}\omega}{g}\psi_{11}^*\,\psi_{11x}(\pm B,0)+*\\&=-hU(\pm B),\end{aligned} \quad \text{(A 4)}$$

where the dependence on t_1 has been suppressed for the sake of brevity. Use has been made of the free-surface condition on ψ_{11} and (4.3). When (6.20) and (A 4) are combined, we find:

$$\int_{S_0^+}\psi_{20x}\,\mathrm{d}z-\int_{S_0^-}\psi_{20x}\,\mathrm{d}z=hU\bigg|_{-B}^{+B}.$$

Because of (A 2) we get

$$\int_{S_\infty^+}\psi_{20x}\,\mathrm{d}z-\int_{S_\infty^-}\psi_{20x}\,\mathrm{d}z=hU\bigg|_{+\infty}^{+B}-hU\bigg|_{-\infty}^{-B}-hU\bigg|_{-B}^{+B}=-hU\bigg|_{-\infty}^{+\infty}, \quad \text{(A 5)}$$

where $U(\pm\infty)$ can be explicitly computed from (6.5):

$$\begin{aligned}ghU(-\infty)&=-\{\mathrm{i}\omega a^*f_0(0)\,[\mathrm{i}ka f_0(0)]+\mathrm{i}\omega R^*a^*f_0(0)\,[-\mathrm{i}kR\,af_0(0)]\}+*\\&=2\omega k|a|^2\,(1-|R|^2)f_0^2(0)\end{aligned} \quad \text{(A 6)}$$

$$\begin{aligned}ghU(\infty)&=-\{\mathrm{i}\omega a^*T^*f_0(0)\,[\mathrm{i}ka\,Tf_0(0)]+*\}\\&=2\omega k\,|a|^2\,|T|^2f_0^2(0).\end{aligned} \quad \text{(A 7)}$$

Since the body is freely floating as far as the high-frequency motion is concerned, energy is conserved:

$$|R|^2+|T|^2=1. \quad \text{(A 8)}$$

Combining (A 6)–(A 8) we get a very simple result:

$$U(+\infty) = U(-\infty) \tag{A 9}$$

which proves (6.22).

Appendix B. The slow radiation potential

In the near field within a few short wavelengths from the body, the rigid-lid assumption applies for the slow potential on the free surface. Let the ambient flow velocity be ϵv, and the slow-sway displacement be of unit amplitude. The sway velocity and acceleration are

$$\mathrm{Re}\,[-2\mathrm{i}\Omega\,\mathrm{e}^{-2\mathrm{i}\Omega t_1}], \quad \mathrm{Re}\,[-4\Omega^2\,\mathrm{e}^{-2\mathrm{i}\Omega t_1}]. \tag{B 1}$$

Let the near-field potential be

$$\psi_{10}^{(2)} = \mathrm{Re}\,[\psi'\,\mathrm{e}^{-2\mathrm{i}\Omega t_1}], \tag{B 2}$$

and the ambient current velocity be

$$v = \mathrm{Re}\,[v'\,\mathrm{e}^{-2\mathrm{i}\Omega t_1}]. \tag{B 3}$$

ψ' has the following outer approximation in the fixed coordinate system:

$$\psi' \to [v'x \pm (v'+2\mathrm{i}\Omega)\,c] \quad (kx \to \pm\infty), \tag{B 4}$$

where c is the blockage coefficient. In the far field, the radiated long-wave potential may be written as

$$\phi_{10}^{(2)} = \mathrm{Re}\,[\phi'\,\mathrm{e}^{-2\mathrm{i}\Omega t_1}], \tag{B 5}$$

where

$$\phi' = \pm\alpha\,\mathrm{e}^{\mathrm{i}k_1|x_1|}, \quad x_1 \lessgtr 0, \tag{B 6}$$

α is an amplitude function and $ghk_1^2 = (2\Omega)^2$. Use has been made of the fact that the slow velocity field due to sway is even in x_1, hence ϕ' is odd in x_1. Note that $\phi_{10}^{(2)}$ is a solution to the long-wave equation.

The inner expansion of ϕ' is

$$\phi' \to \pm\alpha[1+\mathrm{i}k_1|x_1|+O(k_1x_1)^2], \quad x_1 \lessgtr 0. \tag{B 7}$$

Matching the asymptotic expansions (B 4) and (B 7) immediately yields

$$\epsilon\mathrm{i}k_1\alpha = v', \tag{B 8}$$

$$\alpha = (v'+2\mathrm{i}\Omega)\,c, \tag{B 9}$$

from which we find

$$v' = \frac{2\epsilon\Omega c k_1}{\epsilon\mathrm{i}k_1 c - 1}. \tag{B 10}$$

The amplitude of the potential $\psi_{10}^{(2)}$ per unit amplitude of sway displacement is therefore given by

$$\alpha = \frac{-2\mathrm{i}\Omega c}{\epsilon\mathrm{i}k_1 c - 1}. \tag{B 11}$$

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